Solution for final exam - Fall 2010

January 18, 2011

Problem 1

1. Price is given by:

$$P = 4 * (1 + 0.0437)^{-1} + 4 * (1 + 0.0437)^{-2} + 104 * (1 + 0.0437)^{-3} = 98.98$$

2. It is simple to derive the forward rate:

$$(1 + r_2)^2 = (1 + r_1) * (1 + f_{1,2}) \Leftrightarrow f_{1,2} = \frac{(1 + r_2)^2}{(1 + r_1)} - 1$$

Inserting $r_1 = 0.04$ and $r_2 = 0.06$, we find that $f_{1,2} = 8.04\%$. Alternatively find the discount factors and use $f_{1,2} = \frac{d_t}{d_{t+1} - 1}$.

3. The minimum variance portfolio is found by setting $\Omega w_{min} = i$, where i is a vector of ones. As $\Omega w_{min} = i \Leftrightarrow w_{min} = \Omega^{-1} i$, we find

$$w_{min} = \begin{pmatrix} 94.7 & -57.3 & -2.2 \\ 30.8 & 167.4 & -70.5 \\ -19.8 & 35.2 & 116.7 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 35.2 \\ 127.8 \\ 132.2 \end{pmatrix} \tag{1}$$

where Ω^{-1} is found in Excel. The solution is easily normalized to

$$w_{min} = \begin{pmatrix} 0.119 \\ 0.433 \\ 0.448 \end{pmatrix}.$$

4. The use of the two-fund separation theorem allows us to draw the mean-variance frontier. See chart on the last page.

5. A higher β corresponds to a higher risk as the correlation with the market is higher. A leveraged portfolio in the market - for instance with a gearing of 2 - will give the same return as a $\beta = 2$ portfolio.

6. δ is the derivative with respect to the underlying asset, i.e. it measures the sensitivity of the option to changes in the underlying asset.

γ is the second derivative with respect to the underlying asset, i.e. it can be compared to the convexity measure of bonds.

ν is the derivative with respect to the volatility, i.e. it measures the sensitivity of the option to changes in the volatility.

θ is the derivative with respect to time, i.e. it measures the sensitivity of the option to the passage of time.

ρ is the derivative with respect to the risk-free interest rate, i.e. it measures the sensitivity of the option to changes in the risk-free interest rate.

7. For a call, the pay-off function is given by:

$$Pay - off(call) = max(0, S - K)$$

Similarly the pay-off function for a put-option is given by:

$$Pay - off(put) = max(0, K - S)$$

The return on a long call and short put resembles that of a futures/forward contract. Obviously the put-call parity comes into play here, a long call and a short put and long PV(K) (K at exercise time) will replicate exactly the stock price.

8. The Modliagni-Miller theorem on capital structure says the the financing of the company, i.e. the mix of equity and debt in its financing, has no importance of the pricing of the companys value. The theorem hinges on a number of strict assumptions:
i) Perfect capital markets, i.e. shorting and position taking can be done costless.

ii) No taxes and transaction costs

iii) Bankruptcy costs are zero, i.e. any bankruptcy is settled immediately and withot any costs.

Problem 2

1. Inserting $r_f = 5\%$, $r_M = 15\%$, $\beta = 0.5$ and respectively $V = 0\%$ or $V = 2\%$ into the APT equation $r_e = r_f + \beta_1(r_M - r_f) + V$ gives $r_e = 10\%$ and $r_e = 12\%$. The net cashflow is then discounted with the r_e. This gives a NPV(10%)=1.66 and NPV(12%)=-8.50. Problem: It is not clearly defined what whether we start a period 0 or 1. Normal terminology would imply that period 1 is payments in one year, but this is not important for the results, whether you start at period 0 or 1 and full credits will be given in both cases.

2. This problem is probably too difficult, so some leniency in scoring the problem is given. It is therefore important to demonstrate an understanding of the NPV concept. A simple, but not fully correct answer, would state that where the capital cost now becomes $r_c = 9.5\%$, i.e. calculating the WACC. This gives a NPV(10%)=4.40.

A more correct answer would include the calculation of the interest rate payments and a pay-down schedule of the debt.

3. WACC cannot be used - at least not directly, as the debt cashflow is not constant.

4. The cashflow now becomes $\begin{bmatrix} -105 & -2.5 & 22.5 & 48.75 & 61.25 \end{bmatrix}$. What is important is that the expected cashflow now becomes certain, as the second investor guarentees the cashflow. Hence it should be discounted with the riskfree rate of 5%. The NPV(5%)=5.26, hence the proposition does add value - assuming the company recognises this. A discussion of the different discounting rates should be made here, as the risk profile of the project changes.

Problem 3

1. In period 1 a call option and put option is sold at strike 10. If the price moves up to 11, the call option is worth 1 and the put 0. If the price moves down to 9 - the call option pays out 0 and the put option 1. A similar strategy is employed in the second period - if in the up state, a call and put option with strike 11 will give a payoff of 1. If in the down state, a call and put option with strike 9 will give the payoff of 1.

2. As the straddle obviously gives a certain return, such a strategy is completely riskfree. The first period straddle hence has a value of $p = \frac{1}{1.04} = 0.96$. The second period straddle has a price of $p = \frac{1}{1.04^2} = 0.92$, but this depends on knowing which state we ended up in. The important thing is to realize that the return is riskless.

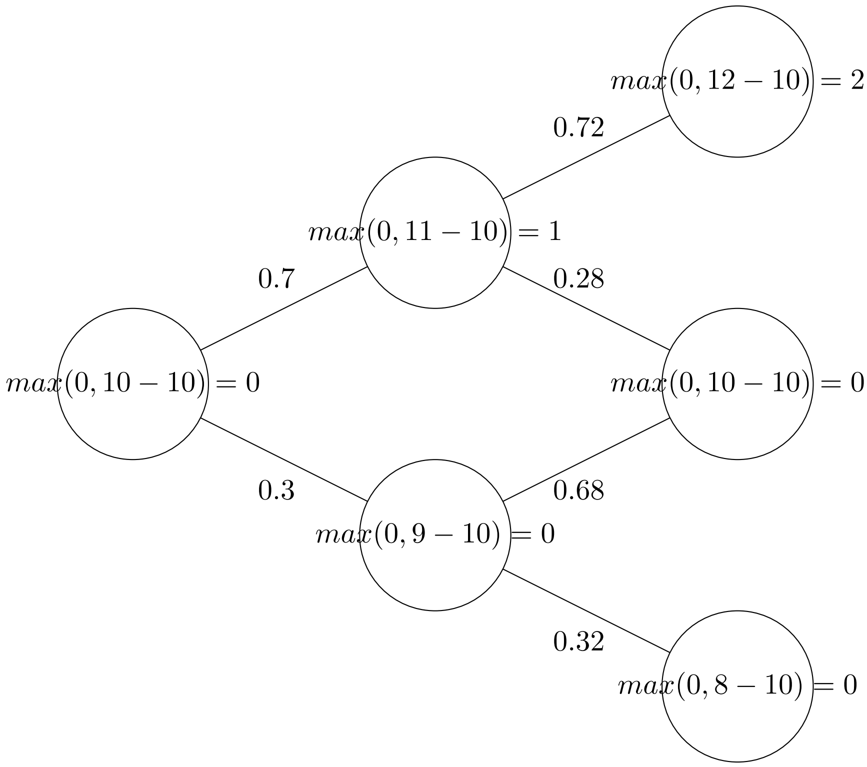
3. We have to find the risk-neutral probabilities.

$$q_u = \frac{1 + r_f - d}{u - d} = \frac{1.04 - 0.9}{1.1 - 0.9} = 0.7$$

and hence

$$q_d = 1 - q_d = 1 - 0.7 = 0.3.$$

This can be used to find $q_{uu} = 0.72$, $q_d = 0.28$, $q_{du} = 0.68$ and $q_{dd} = 0.32$. This can be used to draw the following pay-off diagram.



Using the risk-neutral probabilities and the pay-off in the nodes, the call price in period 1 can be found (ignoring terms with a 0 payoff).

$$c(strike = 10) = 0.7 * 0.72 * 2 * 1.04^{-2} = 0.93.$$

Similarly the put option is priced using the pay-off (2 in the down-down node and 0 otherwise).

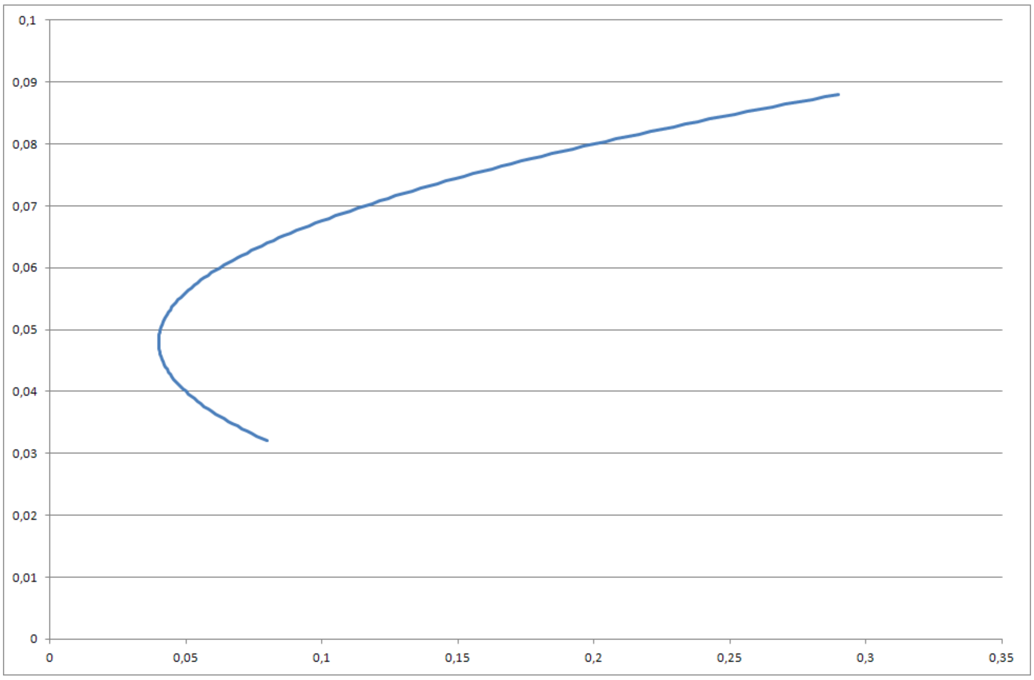
$$p(strike = 10) = 0.3 * 0.32 * 2 * 1.04^{-2} = 0.18.$$

4. The put and call options are priced with the same strike price, same volatility, same risk-free rate and same period. The put-call parity can therefore be used and holds.

Specifically

$$10 * 1.04^{-2} + 0.93 = 10 - 0.18 = 9.82$$

holds. A link to problem 1.7 would be nice.


0,1
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35